Abstract— In the emerging paradigm of interoperal network, the cognitive users are allowed to transmit opportunistically on a temporarily empty frequency band which is authorized to the licensed users. To support this spectrum sharing functionality, the cognitive users dynamically sense the radio frequency environment for being aware of the high-priority licensed users. Spectrum sensing becomes challenging in the wideband regime due to high sampling frequency functioning at or above Nyquist rates. Based on the sparseness of the wideband signal, the spectrum can be recovered with only very few compressive measurements, thus employs relief of high-speed signal processing units. This paper proposes an efficient way for wideband cognitive receiver sensing unit that estimate the highly sparse segment of wideband through compressed sensing rather than entire wideband spectrum and then discover spectral opportunity for a cognitive user. The proposed model deals with the highly-sparse signal segment which provides better spectral estimation and hence improves the detection performance, demonstrated by the simulation. Eventually, reduction of computational complexity as well as a level up of detection performance of the proposed method has sorted out compared to a single RF chain followed by compressive sensing.

Index terms—compressive sensing, $l_1$-norm minimization, analog-to-information converter, wideband spectrum sensing, spectrum estimation.

I. INTRODUCTION

Recent research shows that at any particular spatial region and time, spectrum is often poorly utilized [1–2] by the licensed or primary users (PUs), thus promoting inefficient use of radio resources. This unused spectrum can opportunistically be accessed by a cognitive radio (CR), key of new paradigm of spectrum agile wireless networks which improve overall spectrum usage [3]. The CR with a wider spectral alertness could potentially exploit more spectral opportunities and obtain greater achievable rates for data transmission [4]. Unfortunately, spectrum sensing in the wideband regime faces significant technical challenges and there are several ways to perform this operation [4–5]. First, a bank of tunable narrowband band-pass filters (BPFs) is incorporated at the radio front-end to scan one narrowband frequency at a time. Second, a single wideband radio front-end followed by digital signal processing (DSP) unit can be used to flexibly search over multiple frequency bands simultaneously. In this scheme, high-speed analog-to-digital converter is required to cope with extremely high sampling rates operating at or above Nyquist rates resulting in excessive memory occupation and energy consumption.

Compressive sampling (CS) [6–9] is a method for acquisition of sparse signals by considering a few samples at a rate that is significantly lower than the Nyquist sampling frequency; the problem of signal reconstruction can be solved with convex optimization problem, called $l_1$-norm regularization, that uses either basis pursuit (BP) [10] or some other greedy pursuits [11–12]. These schemes provide an effective way to sense the discrete-time sparse (sparsity in frequency domain) signals and perfectly (or near perfectly) reconstructed with a few number of random samples that comes out from an analog-to-information converter (AIC) or a random demodulator (RD) [13–14]. In [3,14,15] authors are devoted to estimate the whole wideband spectrum to find a spectrum hole for opportunistic access of CRs. Estimating the entire band in CS domain implies computational burden as well as it requires more memory space to store data and hence prohibitive energy cost for new-generation embedded devices.

To overcome those problems, this paper gives emphasis to estimate a significant part (which is highly sparse among the segments of the spectrum) of the whole wideband spectrum thus making computational complexity lower. As soon as the wideband signal undergoes to different BPFs that select the RF band of interest and divide the whole wideband spectrum into several frequency bins (FBs). Since sparsity is one of the fundamental requirements for spectral recovery that has already been proposed in CS theory [6–7], primarily, this paper aims to discover the highly-sparse frequency bin (HSFB) by the average energy comparison of each FBs. The energy estimation of every FB is performed by taking random sub-Nyquist rate samples coming out from the RD. The HSFB exploits several indications; first, it ensures of having minimum number of PUs active which substantially exploit maximum opportunistic accessibility for a CR user. Second, the more the sparsity, the better would be the spectral estimation which pays improved detection performance. Third, spectral estimation of a single HSFB rather than entire wideband would ask lower computational complexity. Now we give emphasis on spectral estimation of the HSFB via a convex optimization approach called $l_1$-norm minimization. Later, we pay attention to check the spectrum occupancy status of a PU by using the energy detector (ED) [16]. Proposed approach outperforms existing wideband spectrum sensing methods, in the sense of lower computational burden, lower memory requirements “in press” [17] as well as higher detection performance. In later sections, we provide some CS basics and signal and system model respectively. Finally, simulation results validate the effectiveness of the proposed model.
II. COMPRESSION SAMPLING

In the CS framework [6-9] a real-valued, finite-length, one-dimensional time-variant signal \( x(t), 0 \leq t \leq x \), can be denoted as a finite weighted sum of orthonormal basis functions (e.g., discrete cosine transform, discrete Fourier transform (DFT), etc.) as follows:

\[
x(t) = \sum_{i=1}^{N} b_i \psi_i(t) = \psi b
\]  

(1)

where only a small number of basis coefficients \( b_i \) signify the sparsity of wideband signal \( x(t) \). Let the acquisition of an \( N \times 1 \) vector \( x = \psi b \) where \( \psi \) is the sparsity basis matrix of size \( N \times N \) and \( b \) an \( N \times 1 \) vector with \( S \), the number non-zero entries in \( b \). In case of sparse signals, an S-sparse depiction of \( x \) can be realized as a linear combination of \( S \) orthonormal basis functions, with \( S \ll N \) and it can be obtained by considering only \( S \) of the \( b_i \) coefficients in (1) that are significant non-zero elements, while the rest \((N-S)\) values representing less significant elements or zeros; this is the basis of the transform coding [7]. Sparsity is influenced by the fact that many natural and man-made signals are compressible in the sense that there exist a set of basis functions, \( \psi \) where the representation of (1) has just a few nonzero elements [6-8]. It is verified that the original signal \( x \) can be recovered using \( M = SO(\log N) \) non-adaptive linear projection measurements onto a measurement matrix \( \Phi \) of size \( M \times N \) which is incoherent with sparsifying basis, \( \psi \) [6-9]. The formation of sensing or measurement matrix \( \Phi \) is given by choosing elements that are drawn independently from a random distribution, e.g. Bernoulli, Gaussian, etc. so the measuring expression, \( y \) can be written as

\[
y = \Phi x = \Phi \psi b = \Theta b
\]  

(2)

where \( \Theta = \Phi \psi \) is a matrix of size \( M \times N \). As \( M \ll N \), the dimension of \( y \) in (2) is much lower than that of \( x \), so there are theoretically infinite solutions to the equation. For these reasons, this problem is ill-posed and the reconstruction of the original signal is quite complex. However, if the condition that \( x \) is \( S \)-sparse is satisfied and with a proper condition of measurement matrix, \( \Phi \) (incoherent with \( \psi \)) and the recovery of \( x \) can be achieved with only \( y \) measurements by solving the minimum \( l_1 \) norm optimization problem [6-7] as follows

\[
\hat{b} = \arg \min_{b} \left\| \Theta b - y \right\|_1 \quad \text{such that} \quad \Theta b = y
\]  

(3)

This is a convex optimization problem that conveniently reduces to a linear program known as basis pursuit (BP) [10], iterative greedy algorithms [11-12], etc. Traditional linear programming techniques can be employed to solve the BP problem whose computational complexities are polynomial in \( N \) [12]. CS method [6] shows that it is possible to recover the original signal (accurately or near accurately) with a much lower sampling ratio than Nyquist rate sampling which outfits with the ADC’s available in the wireless systems.

III. SIGNAL MODEL

Our objective is to decide the PU signal occupancy state of a band of interest within a FB and the band is denoted by \( l \) (\( l = 1, 2, \ldots, L \)). To do so, the test statistic of detecting the occupancy status of PU in a band of interest is measured as \( H_{0,l} \) (absences of a PU) and \( H_{1,l} \) (presence of a PU). That is, we test the following binary hypotheses:

\[
\begin{align*}
X[l] &= \left\{ \begin{array}{ll}
W[l], & H_{0,l} \\
H_n S[l] + W[l], & H_{1,l}
\end{array} \right.
\end{align*}
\]  

(4)

where \( X[l] \) is the spectrum of the band of interest estimated through the promising \( l_1 \)-norm minimization scheme, discussed in [6-7]. \( H_l \) stands for the discrete frequency response between the PU and the CR, \( S[l] \) is the primary signal transmitted within a PU band \( l \) along with complex additive white Gaussian noise (AWGN) \( W[l] \) of zero mean and unity variance.

An energy detector performance does not depend on the a-priori information of PU signal and is less complex to implement [16], thus make it popular in practice, therefore the signal energy is calculated over an interval of \( J \) samples by

\[
E[l] = \sum_{j=0}^{J-1} |\hat{X}_l[l]|^2, \quad l = 1, 2, \ldots, L
\]  

(5)

where \( \hat{X}_l[l] \) indicates the \( j \)-th sub-channel spectral estimation considered by the CR and the decision parameter of the ED is given by

\[
\begin{align*}
H_{0,l} &\quad \Rightarrow \lambda_l, \quad l = 1, 2, \ldots, L \\
H_{1,l} &\quad \Rightarrow \frac{E[l]}{\lambda_l}
\end{align*}
\]  

(6)

where \( \lambda_l \) is the decision threshold of a PU sub-channel of interest inside a FB. Following [16], the signal energy can be labeled as

\[
E[l] = \left( \frac{\chi_{2}^2}{\chi_{2}^2(2\gamma[n])} \right) \Gamma(\gamma[l]), \quad H_{0,l}
\]  

(7)

where \( \gamma[l] \) denotes the signal-to-noise ratio (SNR) at the CR of a frequency band, and \( \chi_{2}^2 \) and \( \chi_{2}^2(2\gamma[n]) \) denote central and non-central chi-square distributions, respectively. Both distributions have degrees of freedom of \( 2 \). For simplicity, we assume that the PU radios deploy uniform power transmission strategy. The probability of detection, \( P_d \) and the probability of false alarm, \( P_{fa} \) can be calculated as in [16]

\[
P_{d,l} = Pr(E[l] > \lambda_l | H_{0,l}) = \frac{I_{\lambda_l}(2\gamma[l])}{\Gamma(\gamma[l])}
\]  

(8)

\[
P_{fa,l} = Pr(E[l] > \lambda_l | H_{1,l}) = Q_{\lambda_l}(\sqrt{2\gamma[l]} \Gamma(\gamma[l]))
\]  

(9)

where, \( \Gamma(u) \) is the gamma function, \( \Gamma(u, x) \) is the incomplete gamma function, and \( Q_{\lambda_l}(u, x) \) denotes the generalized Marcum Q-function.

IV. SYSTEM MODEL AND PROBLEM FORMULATION

The CR receiver is realized through a BPF bank as depicted in Fig. 1. Let the wideband signal \( x_c(t) \) of bandwidth \( W \) Hz is mutually shared among the PUs of a primary network where a limited number of PU sub-channels are randomly available to the CRs for opportunistic accessing. The wideband filter prior to the BPF in Fig. 1 preserves the bandwidth \( W \) of each particular considered sub-band. Let the bandwidth of non-overlapped PU be equal to \( B \) Hz and the same bandwidth can opportunistically serve the necessity of a CR node for data transmission. Hence, the possibilities of maximum number of sub-channels are to be defined as \( \frac{W}{B} \). Let the CR receiver has accommodated \( K \) number of identical BPFs, the outputs of the BPFs are denoted \( x_k \) having identical bandwidth e.g., \( w_k = \frac{W}{K} \) Hz where \( k = 1, 2, \ldots, K \). The wideband signal is then
injected into the filter bank \( \{H_k(f)\}_{k=1}^{K} \) as \( K \ll L \) where \( H_k(f) \) represents the transfer function of the \( k \)-th filter. At any particular time each \( x_k \) is accommodated with different number of active PUs from a maximum of \( w_k/B \) or \( L/K \) PU sub-bands while our goal is to discover HSFB through average energy estimation and classification. To make the problem simpler, we assume that at least one PU sub-channel present in a FB \( x_k \) at a certain time.

The output of the \( k \)-th BPF is then sent to the RD as described in the paper [13] in order to obtain \( M_K \) randomized samples from the \( k \)-th FB. Those \( M_K \) samples are intended for calculating the average energy \( E_k \) of a single FB and compare those average energies \( \{E_k\}_{k=1}^{K} \) at the energy estimate and compare block which is the crucial part in Fig. 1. Intuitively, the reliability of the energy comparison depends on the number of samples taken into account; the more the number of samples to be considered, the better would be the reliability to estimate the average energy of each FB. Therefore, higher compression ratio \( M/N \) (larger number of samples from RD) is anticipated for estimating average energy of FBs. Let us consider equal compression ratio for all FBs to compute the average energies. While comparing the average energy \( E_k \) of a bunch of samples corresponding to every single FB of bandwidth \( w_k \), the comparator also restore the sample values. In comparator, when classifying the HSFB along with the samples of having minimum average energy, \( E_k(\text{min}) \) is considered to estimate the spectral magnitude \( \hat{X}_k \) of HSFB generated via a well-known \( l_1 \)-minimization algorithm [7]-[8].

Later, we exploit a well-known ED [16] approach to search for a PU sub-channel available for opportunistic use by a CR. Now, the CR can resolve the PU occupancy status within the HSFB by using the ED test statistics as of (9); the decision of spectrum sensing regarding the sub-channel of interest.

### Figure 1. Schematic illustration of the filter-based spectrum estimation via compressive sensing

#### A. Computational complexity of the proposed method

Now we try to analyze the computational complexity of this CR receiver block expressed in Fig. 1. As the subsampled Fourier matrix (it is customized by pooling of \( m \) rows selected uniformly at random from the DFT matrix) is applied to the signal recovery so it requires \( O(N\log N) \) operation (precisely), the computational burden is equal to the no. of iterations \( \times N \times \log N \), where no. of iterations is not usually easy to bound, but in worst-case, it can be bounded by \( N \). By using K number of filters, the computational complexity is reduced in the order of \( O(K\log K) \) and it requires \( O\left(\frac{N}{K}\log\frac{N}{K}\right) \).

Besides, we have to take care of estimating the average energy of each FB in a static manner which is in the order of \( O(2\alpha - 1) \approx O(2\alpha) \) where \( \alpha = M/N \). To set large \( \alpha \) exploits better estimation of suitable FB and let \( O(2\alpha) = O(P) \). There is one additional term to work out, used for comparison of the average energy that depends on the number of filters \( O(K) \). As a result, in the proposed method the computational burden is in the order of

\[
\Omega = O\left(P + K + \frac{N}{K}\log\frac{N}{K}\right) \approx O\left(P + \frac{N}{K}\log\frac{N}{K}\right)
\]

where \( K \ll P \) and \( K \ll N \). A detail study of complexity order deviation with no. of BPFs be clarified in Fig. 2 “in press” [17].

Another important entity is to notice the memory space needed for the proposed CR receiver sensing block; there are two terms to consider usually \( O(N) \) bits of memory spaces to be required for the recovered spectrum of length \( N \) and the later is \( O(M \times N) \) for the measurement matrix to store [9]. However, memory space requirement is greatly reduced by the sensing matrix as in the proposed method the space requirement is divided by the \( K \)-th square of \( O(M \times N) \) i.e. \( O\left(\frac{M \times N}{K^2}\right) \). However, we have to spend a few static memory spaces due to the average energy estimation of the random samples comes out from the RD which is at the order of \( O\left(\frac{1}{2}P\right) \) and this term depends on the compression ratio \( M/N \) considered for the average minimum energy \( E_k(\text{min}) \). Hence, the expression of the total memory spaces required for the proposed method is

\[
Y = O\left(\frac{1}{2}P + \frac{N}{K} + \frac{MN}{K^2}\right)
\]

which is greatly influenced by the number of filters. It shows similar characteristics with the computational burden and the analytical figure is not provided here due to page limit.

As computational burden decreases with the increasing number of filters \( K \) and so this does not necessarily mean that high values of \( K \) always increase the sparsity in some basis. If \( K \) is excessively large, the sparsity is reduced in substantial order and hence spectral recovery would be ambiguous to resolve. Thus, selection of higher values of \( K \) have two complications; one is budget constraints for designing such type of CR receiver and another is too high value of \( K \) do not convey suitable sparsity. Therefore, there should be a trade-off to choose the value of \( K \) where sparsity and cost find a best possible way out.

### V. PERFORMANCE ANALYSIS AND SIMULATION RESULTS

We consider, at baseband, the wideband signal \( x_c(t) \) falls in the range of \([1,64]\Delta Hz\) that can accommodate a maximum of 32 non-overlapping PU sub-bands whose bandwidths \( B \) is set to \( 2\Delta Hz \) each and encoded as \( \{\chi_l\}_{l=1}^{32} \), where \( \Delta \) is the frequency resolution. The received signal \( x_c(t) \) before entering the \( K \)-th BPF is as follows

\[
x_c(t) = \sum_{n=1}^{N} \sqrt{E_n \cdot \text{sinc}(B_n(t - \vartheta)) \cdot \cos(\theta_n(t - \vartheta)) + z(t)}
\]

where \( \text{sinc}(x) = \frac{\sin(x)}{\pi x} \), \( \vartheta \) denotes a random time offset with-
in sampling branches, \( \theta_k \) is the angular frequency of \( f_k \). \( z(t) \) is the AWGN as we adopt the received RF signal is degraded by this type of noise of unit variance. In simulations, the number of BPFs are considered as \( K = 4 \) so the bandwidth of \( x_k \) is \( \omega_k = 16\Delta f \), i.e. a single FB can comprise a maximum of \( \omega_k / \Delta f = 8 \) PUs with no sparsity. While observed the burst of transmissions in the network, there are a total of 15 non-overlapping PUs with different carrier frequencies \( \{ f_{n+1} \} \) inside the wideband \( W \) such that the number of active PUs in various FBs as \( \{ x_k \} \) with unlike sparsity levels. The energy comparator selects \( x_4 \) as HSFB tailored for spectral estimation. The number of Nyquist rate samples \( N \) are taken into account from HSFB for an observation time \( T \) (e.g., the chosen frequency resolution \( \Delta f = 1 \text{ MHz} \) and \( N = 1024 \) samples to satisfy \( T = 32\mu s \)). For energy estimation of every single FB, it is chosen a fixed compression ratio \( M / N \) of 40% however, in order to evaluate detection performance, the compression ratio \( M / N \) is set to vary from 1% to 40% aimed at spectral estimation of HSFB using \( l_1 \)-norm minimization. Typically, in wideband radio signals, the DFT has been preferred as the sparsifying basis to assemble the measurement matrix \( \Phi \).

Later, detection performance (Fig. 3) has been tested to a band of interest which is accessible to a CR. Moreover, detection performance of the same RF band of interest is compared with spectral estimation of the wideband signal \( x_k(t) \) preceded over a single RF chain. In that case, number of samples \( N \) has modified accordingly to fix the sampling time \( T = 32\mu s \). Fig. 3 illustrates the effect of the compression ratio \( M / N \) on the wideband spectrum sensing performance by setting probability of false alarm \( P_{fa} = 0.01 \) in (8) and the plot illustrates better performance for the proposed scheme. At the CR node, the received SNR of the active channels are set to 5dB and 10dB. In simulation, the probability of detection \( P_d \) is selected by statistical averaging of 2000 experimental results.

VI. CONCLUSION

In this paper, we have proposed a model of the CR receiver for wideband spectrum sensing via CS. Starting with a time domain wideband signal, the HSFB has been classified and estimated. Since spectral estimation has performed of a unique FB rather than full wideband, in view of that, the proposed method requires less computational complexity. Moreover, the spectral estimation is improved as sparsity increases, which provides the enhanced probability of detection, investigated through simulations to a band of interest of a CR.

**REFERENCES**


